SAP2000® v25

Powered by SAPFIRE®

Structural Analysis and Design

Concrete Shell Design Manual

Eurocode EC2-2004

A PRODUCT OF COMPUTERS & STRUCTURES, INC.
Proudly developed in the United States of America

Concrete Shell Design Manual

Eurocode 2-2004

For

SAP2000®

ISO SAP022123M54 Rev. 1
Proudly developed in the United States of America
January 2024

COPYRIGHT

Copyright © Computers and Structures, Inc., 1978-2024
All rights reserved.

The CSI Logo® and SAP2000® are registered trademarks of Computers and Structures, Inc. Watch & Learn™ is a trademark of Computers and Structures, Inc. Windows is a registered trademark of Microsoft Corporation. Adobe and Acrobat are registered trademarks of Adobe Systems Incorporated.

The computer program SAP2000® and all associated documentation are proprietary and copyrighted products. Worldwide rights of ownership rest with Computers and Structures, Inc. Unlicensed use of this program or reproduction of documentation in any form, without prior written authorization from Computers and Structures, Inc., is explicitly prohibited.

No part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior explicit written permission of the publisher.

Further information and copies of this documentation may be obtained from:

Computers and Structures, Inc.
www.csiamerica.com

info@csiamerica.com (for general information)
support@csiamerica.com (for technical support)

DISCLAIMER

CONSIDERABLE TIME, EFFORT AND EXPENSE HAVE GONE INTO THE DEVELOPMENT AND TESTING OF THIS SOFTWARE. HOWEVER, THE USER ACCEPTS AND UNDERSTANDS THAT NO WARRANTY IS EXPRESSED OR IMPLIED BY THE DEVELOPERS OR THE DISTRIBUTORS ON THE ACCURACY OR THE RELIABILITY OF THIS PRODUCT.

THIS PRODUCT IS A PRACTICAL AND POWERFUL TOOL FOR STRUCTURAL DESIGN. HOWEVER, THE USER MUST EXPLICITLY UNDERSTAND THE BASIC ASSUMPTIONS OF THE SOFTWARE MODELING, ANALYSIS, AND DESIGN ALGORITHMS AND COMPENSATE FOR THE ASPECTS THAT ARE NOT ADDRESSED.

THE INFORMATION PRODUCED BY THE SOFTWARE MUST BE CHECKED BY A QUALIFIED AND EXPERIENCED ENGINEER. THE ENGINEER MUST INDEPENDENTLY VERIFY THE RESULTS AND TAKE PROFESSIONAL RESPONSIBILITY FOR THE INFORMATION THAT IS USED.

Contents

1 Introduction

The design of concrete shell in accordance with the "Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings" (EN 1992-1-1, 2004) and "Eurocode 2: Design of concrete structures – Part 2: Concrete bridges – Design and detailing rules" (EN 1992-2, 2005) is seamlessly integrated within the program. Initiation of the design process, along with control of various design parameters, is accomplished using the Design menu. Automated design at the object level is available for this design code, as long as the structures have first been modeled and analyzed by the program. Model and analysis data, such as material properties and member forces, are recovered directly from the model database, and are used in the design process in accordance with the user defined or default design settings. As with all design applications, the user should carefully review all of the user options and default settings to ensure that the design process is consistent with the user's expectations.

The default implementation in the software is the CEN version of the code. Additional country specific National Annexes are also included. The Nationally Determined Parameters are noted in this manual with [*NDP*]. Changing the country in the Design Preferences will set the Nationally Determined Parameters for the selected country as defined in Appendix B.

It is important to read this entire manual before using the design algorithms to become familiar with any limitations of the algorithms or assumptions that have been made.

For referring to pertinent sections of the corresponding code, a unique prefix is assigned for each code.

- Reference to the EN 1992-1-1:2004 code is identified with the prefix "EC2-1."
- Reference to the EN 1992-2:2005 code is identified with the prefix "EC2-2."

1.1 Units

The EC2-1 and EC2-2 design codes are based on Newton, millimeter, and second units and, as such, so is this manual, unless noted otherwise. Any units, imperial, metric, or MKS may be used in the software in conjunction with Eurocode 2 design.

1.2 Concrete Shell Design

Concrete shell design consists of calculating the amount of reinforcement in the membrane layers required to resist bending moments and axial forces, and shear demand-over-capacity ratio at the nodes of the shell member. The design results at other locations within the shell elements are averaged from those at the nodes.

Program output can be presented graphically on the model and in tables for both input and output data. For each presentation method, the output is in a format that allows the engineer to quickly study the stress conditions that exist in the structure, and in the event the member is not adequate, aid the engineer in taking appropriate remedial measures.

2 Design Algorithms

This chapter provides an overview of the basic assumptions, design preconditions, and some of the design parameters that affect the design of concrete shell.

2.1 Design Capability

The program has the ability to check for concrete cracking of the shell member, design for the amount of longitudinal reinforcement in the membrane layers required to resist bending moments and axial forces, check for shear demand-over-capacity ratio, and design for the amount of transverse reinforcement to resist shear forces at the nodes of the shell member**.** The design algorithm including iteration to determine the thickness of the outer layers and the optimum amount of reinforcement follows the procedure described in Brondum-Nielsen (1974), Colombo et al. (2014), and Craveiro et al. (2021). The shear design with truss model approach and the associated increase in membrane forces is derived from Marti (1990.)

3 Design Process

This chapter provides a detailed description of the algorithms used by the programs in the design/check of structures in accordance with "Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings" (EN 1992-1-1, 2004) and "Eurocode 2: Design of concrete structures – Part 2: Concrete bridges – Design and detailing rules" (EN 1992-2, 2005.)

3.1 Notations

The various notations used in this chapter are described herein.

a^t, a^b	Thickness of the top and bottom layers, respectively, mm
A_{sw1}, A_{sw2}, A_{sw}	Area of transverse reinforcement per unit length in direction 1 and 2, and principal shear direction, respectively, mm^2/mm
A_{s1}^t, A_{s1}^b	Area of reinforcement per unit length of the top and bottom layers along direction 1, respectively, mm^2/mm
A_{s2}^t, A_{s2}^b	Area of reinforcement per unit length of the top and bottom layers along direction 2, respectively, mm^2/mm
f_c	Concrete compressive strength of the outer layer, N/mm^2
f_{cd}	Design value of concrete compressive strength, N/mm^2
f_{cd1}	Compressive strength of uncracked concrete, N/mm^2
f_{cd2}	Compressive strength of cracked concrete, N/mm^2
f_{ck}	Characteristic compressive cylinder strength of concrete at 28 days, N/mm^2
f_{yd}	Design yield strength of reinforcement, N/mm^2
f_{yk}	Characteristic yield strength of reinforcement, N/mm^2
h	Thickness of the shell element, mm

h^t, h^b	Distance from the center of the shell element to the center of the top and bottom layers, respectively, mm
h_{s1}^t, h_{s1}^b	Distance from the center of the shell element to the center of the reinforcement in the top and bottom layers along direction 1, respectively, mm
h_{s2}^t, h_{s2}^b	Distance from the center of the shell element to the center of the reinforcement in the top and bottom layers along direction 2, respectively, mm
M_{11}, M_{22}, M_{12}	Resultant moments per unit length of the shell element, N/mm
N_{11}, N_{22}, N_{12}	Resultant axial forces and in-plane shear force per unit length of the shell element, respectively, N/mm
N_{11}^t, N_{11}^b	Axial force component per unit length along direction 1 at the center of the top and bottom layers, respectively, N/mm
N_{22}^t, N_{22}^b	Axial force component per unit length along direction 2 at the center of the top and bottom layers, respectively, N/mm
N_{12}^t, N_{12}^b	Shear force component per unit length at the center of the top and bottom layers, respectively, N/mm
N_c^t, N_c^b	Principal compressive force per unit length at the center of the top and bottom layers, respectively, N/mm
N_{max}^t, N_{max}^b	Maximum principal force per unit length at the center of the top and bottom layers, respectively, N/mm
N_{min}^t, N_{min}^b	Minimum principal force per unit length at the center of the top and bottom layers, respectively, N/mm
N_{s11}^t, N_{s11}^b	Axial tensile force component per unit length along direction 1 at the center of the top and bottom layers to be resisted by the top and bottom reinforcement, respectively, N/mm
N_{s22}^t, N_{s22}^b	Axial tensile force component per unit length along direction 2 at the center of the top and bottom layers to be resisted by the top and bottom reinforcement, respectively, N/mm
N_{d11}^t, N_{d11}^b	Design axial force component per unit length along direction 1 at the reinforcement location and to be resisted by the top and bottom reinforcement, respectively, N/mm
N_{d22}^t, N_{d22}^b	Design axial force component per unit length along direction 2 at the reinforcement location and to be resisted by the top and bottom reinforcement, respectively, N

s	Spacing of transverse shear reinforcement, mm
$S_{11}^{t}, S_{22}^{t}, S_{12}^{t}$	Axial stresses in direction 1 and 2, and in-plane shear stress at the top of the shell element, respectively, N/mm^2
$S_{11}^{b}, S_{22}^{b}, S_{12}^{b}$	Axial stresses in direction 1 and 2, and in-plane shear stress at the bottom of the shell element, respectively, N/mm^2
$S_{11}^{m}, S_{22}^{m}, S_{12}^{m}$, S_{13}^{m}, S_{23}^{m}	Axial stresses in direction 1 and 2, in-plane shear stress, and transverse shear stresses at the middle of the shell element, respectively, N/mm^2
V_1, V_2	Resultant transverse shear forces per unit length of the shell element in direction 1 and 2, respectively, N/mm
V_o	Resultant transverse shear force per unit length of the shell element in principal shear direction, N/mm
$V_{Rd,c}$	Concrete shear capacity per unit length of the shell element in principal shear direction, N/mm
$V_{Rd,max}$	Maximum shear capacity per unit length of the shell element in principal shear direction, N/mm
α_{CC}	Coefficient taking account of long-term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied
α_{CT}	Coefficient taking account of long-term effects on the tensile strength and of unfavorable effects resulting from the way the load is applied
ε_{c3}	Strain at concrete compressive strength f_{cd}
ε_{yd}	Yield strain of reinforcement
θ	Angle between 1-direction and the principal tensile direction (perpendicular to the cracks)
φ_0	Angle between 1-direction and the principal shear direction
γ_C	Partial factor for concrete
γ_S	Partial factor for reinforcement

3.2 Design Load Cases and Combinations

The shell structure is to be designed based on the load cases and/or combinations defined by the users. For multi-step load cases or combinations with envelope option selected for output, the design is carried out for the set of forces/moments at each step, and the final design results are enveloped over all the steps. If the load case or combination has double values with maximum and minimum in the output such as the case with response spectrum, steady state, or power-spectral density, the maximum and minimum values of the membrane force components N_{11}, N_{22}, and N_{12} (as described in the next section) are determined and the design is performed for eight (8) permutations of these membrane forces.

For the load combination defined with envelope combination type, the design is carried out for each set of forces/moments at each step of each sub-level load case (or combination) within the defined load combination, and the final design results are enveloped over all the steps of all the sub load cases (or combinations). If the sub-level load combination is defined with envelope type, the same procedure as described precedingly is done for this sub-level load combination.

3.3 Cracked/Uncracked Shell

The shell elements are checked for cracking condition at three (3) different levels (i.e., top, middle, and bottom) as follows:

$$\Phi = \alpha \frac{J_2}{f_{cd}^2} + \lambda \frac{\sqrt{J_2}}{f_{cd}} + \beta \frac{I_1}{f_{cd}} - 1 \begin{cases} \leq 0 & Uncracked \\ > 0 & Cracked \end{cases} \quad \text{(EC2-2 Eq. LL.101)}$$

where:

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \quad \text{(EC2-2 Eq. LL.102)}$$

$$J_3 = (\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m) \quad \text{(EC2-2 Eq. LL.103)}$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad \text{(EC2-2 Eq. LL.104)}$$

$$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad \text{(EC2-2 Eq. LL.105)}$$

$$\alpha = \frac{1}{9k^{1.4}} \quad \text{(EC2-2 Eq. LL.106)}$$

$$\lambda = \begin{cases} c_1 cos\left[\frac{1}{3} arcos(c_2 cos3\theta)\right] & \text{for } cos3\theta \geq 0 \\ c_1 cos\left[\frac{\pi}{3} - \frac{1}{3} arcos(-c_2 cos3\theta)\right] & \text{for } cos3\theta < 0 \end{cases} \quad \text{(EC2-2 Eq. LL.107)}$$

$$\beta = \frac{1}{3.7k^{1.1}} \quad \text{(EC2-2 Eq. LL.108)}$$

$$cos3\theta = \frac{3\sqrt{3}}{2} \frac{J_3}{{J_2}^{3/2}} \quad \text{(EC2-2 Eq. LL.109)}$$

$$c_1 = \frac{1}{0.7k^{0.9}} \quad \text{(EC2-2 Eq. LL.110)}$$

$$c_2 = 1 - 6.8(k - 0.07)^2 \quad \text{(EC2-2 Eq. LL.111)}$$

$$k = \frac{f_{ctd}}{f_{cd}} \quad \text{(EC2-2 Eq. LL.112)}$$

f_{cd} and f_{ctd} are the design value of compressive and tensile strength of concrete. They are used to calculate Φ and k instead of f_{cm} and f_{ctm} as described in EC2-2 Eq. LL.101 and LL.112.

f_{cd} and f_{ctd} are determined as follows:

$$f_{ctd} = \alpha_{CT} \frac{f_{ctk,0.05}}{\gamma_C}$$

$$f_{cd} = \alpha_{CC} \frac{f_{ck}}{\gamma_C}$$

$$f_{ctd} = 0.7 f_{ctm}$$

$$f_{ctm} = \begin{cases} 0.30 f_{ck}^{2/3} & f_{ck} \leq 50\ MPa \\ 2.12 ln\left(1 + \frac{f_{cm}}{10}\right) & f_{ck} > 50\ MPa \end{cases}$$

$$f_{cm} = f_{ck} + 8\ (MPa)$$

f_{ck} = characteristic compressive cylinder strength of concrete at 28 days

γ_C = partial factor for concrete

α_{CC} = coefficient taking account of long-term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied

α_{CT} = coefficient taking account of long-term effects on the tensile strength and of unfavorable effects resulting from the way the load is applied

σ_1, σ_2, and σ_3 are the principal stresses at the three levels considered and derived from the following stress tensors:

Top level: $$\begin{bmatrix} S_{11}^t & S_{12}^t & 0 \\ S_{12}^t & S_{22}^t & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{Bmatrix} \sigma_1^t \\ \sigma_2^t \\ \sigma_3^t = 0 \end{Bmatrix}$$

Middle level: $$\begin{bmatrix} S_{11}^m & S_{12}^m & S_{13}^m \\ S_{12}^m & S_{22}^m & S_{23}^m \\ S_{13}^m & S_{23}^m & 0 \end{bmatrix} \rightarrow \begin{Bmatrix} \sigma_1^m \\ \sigma_2^m \\ \sigma_3^m \end{Bmatrix}$$

Bottom level: $$\begin{bmatrix} S_{11}^b & S_{12}^b & 0 \\ S_{12}^b & S_{22}^b & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{Bmatrix} \sigma_1^b \\ \sigma_2^b \\ \sigma_3^b = 0 \end{Bmatrix}$$

For the middle level, the shear stresses S_{13}^m and S_{23}^m are determined by the Jourawski method:

$$S_{13}^m = 1.5 \frac{V_1}{h}$$

$$S_{13}^m = 1.5 \frac{V_2}{h}$$

where V_1 and V_2 are the resultant shear forces on the shell element in directions 1 and 2, respectively, and h is the thickness of the shell element.

In case the shell element is uncracked, the design is satisfied. Nothing else needs to be checked even the principal stress because the design strength of concrete, f_{cd} and f_{ctd}, are used calculate Φ and k instead of f_{cm} and f_{ctm}.

For the case in which shell element is determined as cracked, the design for longitudinal and transverse reinforcement is described in the subsequent sections.

3.4 Three-Layer (Sandwich) Design Model

In general, shell elements are subjected to eight stress resultants. Those are the three membrane force components N_{11}, N_{22}, and N_{12}; the two flexural moment components M_{11} and M_{22} and the twisting moment M_{12}; and the two transverse shear force components V_1 and V_2 (Figure 3-1.) For the purpose of design, the shell is isolated as a unit element. It is further idealized as comprising two outer layers and an uncracked core – this is sometimes called a "sandwich model." The outer layers of the sandwich model are assumed to carry moments and membrane forces, while the transverse shear forces are assigned to the core as shown in Figure 3-2. The dimensions of the sandwich layers and the location of the reinforcement are illustrated in Figure 3-3. The design implementation in the software assumes there are no diagonal cracks in the core. In such a case, a state of pure shear develops within the core, and hence the transverse shear force at a section has no effect on the in-plane forces in the sandwich covers. Thus, no transverse reinforcement needs to be provided, and the in-plane reinforcement is not enhanced to account for transverse shear.

It is important to note that the design will not be performed for the case in which the sum of the concrete covers of the top and bottom reinforcement exceeds 95% of the shell section thickness. It is to avoid any problem associated with numerical difficulties in calculation of the forces in the reinforcement at its actual location.

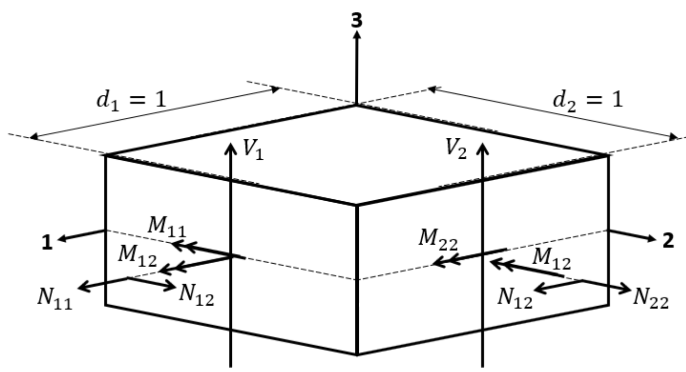

Figure 3-1: Unit shell element and stress resultants

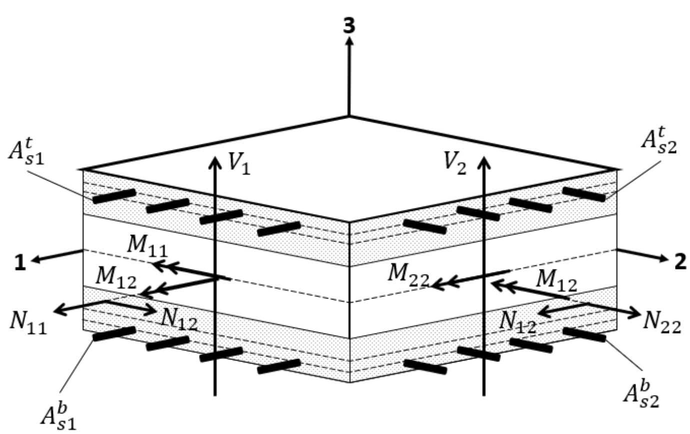

Figure 3-2: Idealized sandwich model

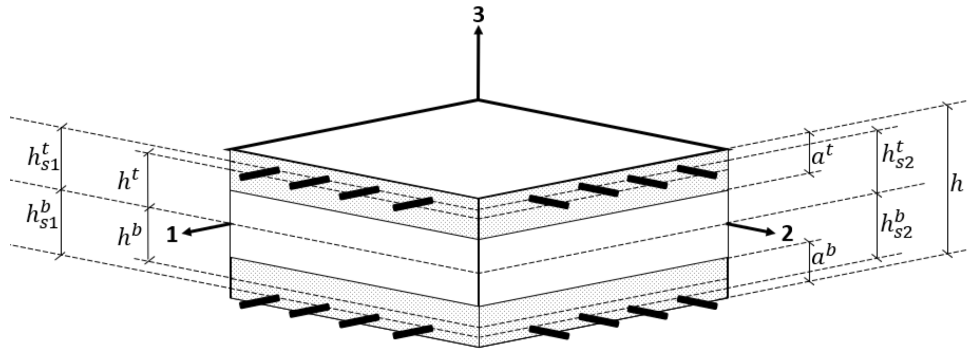

Figure 3-3: Dimensions of the sandwich model

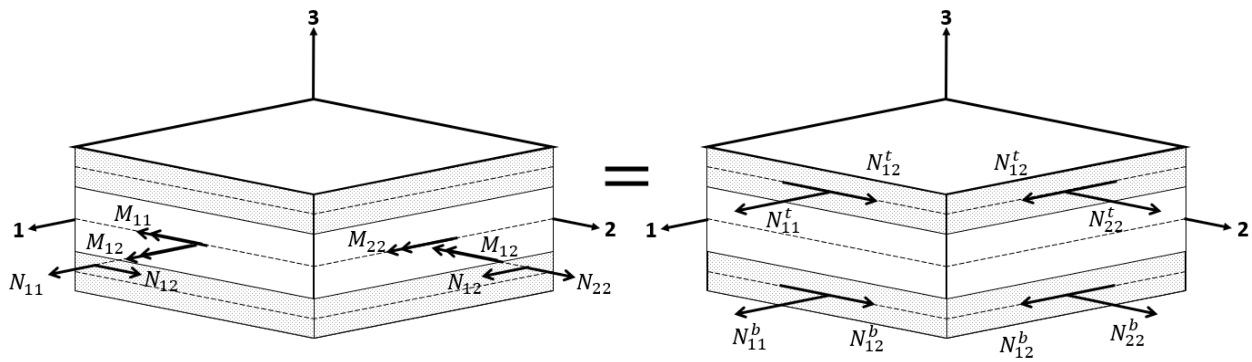

Figure 3-4: Loads on the outer layers of the sandwich model

3.4.1 Design for Axial Forces and Bending Moments

The membrane forces of the top and bottom layers of the sandwich with respect to the center of the layers as shown in Figure 3-4 are determined as follows:

$$\sum M_{center\ of\ the\ bottom\ layer} = 0 \rightarrow N_{11}^t = \frac{N_{11}h^b - M_{11}}{h^t + h^b}$$

$$\sum M_{center\ of\ the\ bottom\ layer} = 0 \rightarrow N_{22}^t = \frac{N_{22}h^b - M_{22}}{h^t + h^b}$$

$$\sum M_{center\ of\ the\ bottom\ layer} = 0 \rightarrow N_{12}^t = \frac{N_{12}h^b - M_{12}}{h^t + h^b}$$

$$\sum M_{center\ of\ the\ top\ layer} = 0 \rightarrow N_{11}^b = \frac{N_{11}h^t + M_{11}}{h^t + h^b}$$

$$\sum M_{center\ of\ the\ top\ layer} = 0 \rightarrow N_{22}^b = \frac{N_{22}h^t + M_{22}}{h^t + h^b}$$

$$\sum M_{center\ of\ the\ top\ layer} = 0 \rightarrow N_{12}^b = \frac{N_{12}h^t + M_{12}}{h^t + h^b}$$

For the top (or bottom) membrane element as shown in Figure 3-5(a), the forces required to be resisted by the reinforcement (N^t_{s11} and N^t_{s22}) and concrete (N^t_c) at the center of the layer are determined as follows:

In Figure 3-5(b): $\sum F_1 = 0 \rightarrow N^t_{s11} = N^t_{11} + N^t_{12} tan\theta$

$\sum F_2 = 0 \rightarrow N^t_{s22} = N^t_{22} + N^t_{12} cot\theta$

In Figure 3-5(c): $\sum F_1 = 0 \rightarrow N^t_c = -\frac{N^t_{12}}{sin\theta cos\theta}$

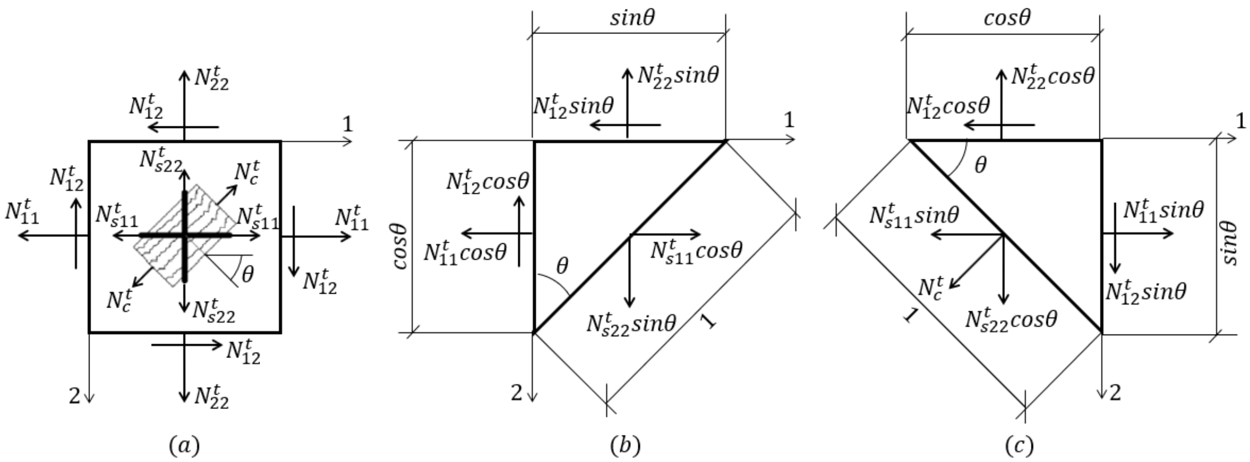

Figure 3-5: Top layer: (a) forces per unit length; (b) section parallel to the crack; (c) section perpendicular to the crack

For design case I in which the reinforcement is required in both directions, $\theta = 45°$ provides the most economical amount of reinforcement and results in:

$$N^t_{s11} = N^t_{11} + |N^t_{12}|$$
$$N^t_{s22} = N^t_{22} + |N^t_{12}|$$
$$N^t_c = -2|N^t_{12}|$$

If $N^t_{s11} < 0$ or $N^t_{11} < -|N^t_{12}|$, which is design case II, the reinforcement in direction 1 is not required. Assuming $N^t_{s11} = 0$:

$$tan\theta = -\frac{N^t_{11}}{N^t_{12}}$$
$$N^t_{s22} = N^t_{22} - \frac{(N^t_{12})^2}{N^t_{11}}$$
$$N^t_c = N^t_{11} + \frac{(N^t_{12})^2}{N^t_{11}}$$

If $N^t_{s22} < 0$ or $N^t_{22} < -|N^t_{12}|$, which is design case III, the reinforcement in direction 2 is not required. Assuming $N^t_{s22} = 0$:

$$tan\theta = -\frac{N^t_{12}}{N^t_{22}}$$

$$N_{s11}^{t} = N_{11}^{t} - \frac{(N_{12}^{t})^2}{N_{22}^{t}}$$
$$N_{c}^{t} = N_{22}^{t} + \frac{(N_{12}^{t})^2}{N_{22}^{t}}$$

If both $N_{s11}^{t} < 0$ and $N_{s22}^{t} < 0$, which is design case IV, the reinforcement in both direction is not required. In this case, the value of N_{c}^{t} is the minimum principal force N_{min}^{t}.

The principal forces are obtained by:

$$N_{max}^{t} = \frac{N_{11}^{t} + N_{22}^{t}}{2} + \sqrt{\left(\frac{N_{11}^{t} - N_{22}^{t}}{2}\right)^2 + (N_{12}^{t})^2}$$
$$N_{min}^{t} = \frac{N_{11}^{t} + N_{22}^{t}}{2} - \sqrt{\left(\frac{N_{11}^{t} - N_{22}^{t}}{2}\right)^2 + (N_{12}^{t})^2}$$

The forces N_{s11}^{t} and N_{s22}^{t} are calculated at the center of the layer. The forces in the reinforcement at its actual location is determined as illustrated in Figure 3-6:

Case (a) – reinforcement required in both top and bottom layer:

$$\sum M_{bottom\ reinforcement} = 0 \rightarrow N_{d11}^{t} = \frac{N_{s11}^{t}\left(h^{t} + h_{s1}^{b}\right) + N_{s11}^{b}\left(h_{s1}^{b} - h^{b}\right)}{\left(h_{s1}^{t} + h_{s1}^{b}\right)}$$
$$N_{d11}^{b} = N_{s11}^{t} + N_{s11}^{b} - N_{d11}^{t}$$

Case (b) – reinforcement required in top layer only:

$$\sum M_{center\ of\ the\ bottom\ layer} = 0 \rightarrow N_{d11}^{t} = \frac{N_{s11}^{t}\left(h^{t} + h^{b}\right)}{\left(h_{s1}^{t} + h^{b}\right)}$$
$$\Delta N_{11}^{b} = N_{s11}^{t} - N_{d11}^{t}$$

Case (c) – reinforcement required in bottom layer only:

$$\sum M_{center\ of\ the\ top\ layer} = 0 \rightarrow N_{d11}^{b} = \frac{N_{s11}^{b}\left(h^{t} + h^{b}\right)}{\left(h_{s1}^{b} + h^{t}\right)}$$
$$\Delta N_{11}^{t} = N_{s11}^{b} - N_{d11}^{b}$$

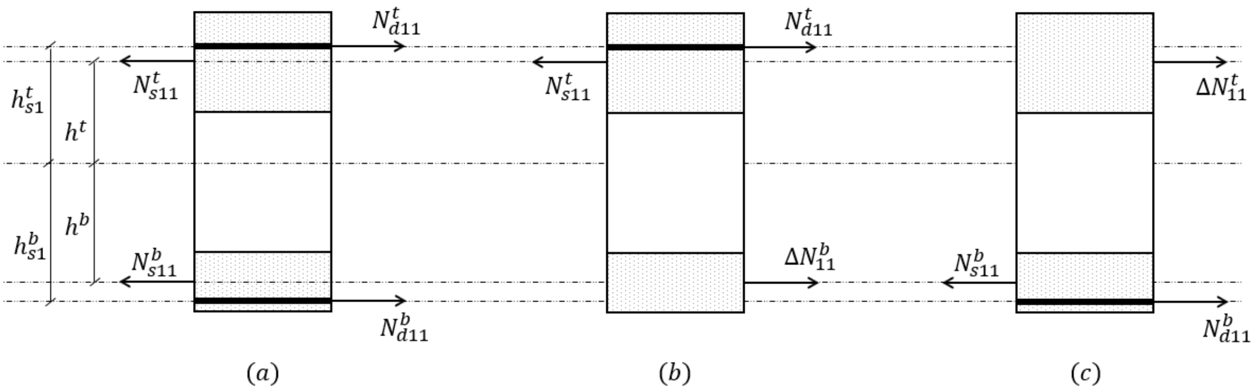

Figure 3-6: Loads on the reinforcement for case in which the reinforcement is required in: (a) both top and bottom layers; (b) only top layer; (c) only bottom layer

The forces in the reinforcement along direction 2 are determined similarly.

The required area of reinforcement per unit length and the design thickness of the top and bottom layers are then computed as:

$$A_{s1}^t = \frac{N_{d11}^t}{f_{yd}}$$

$$A_{s2}^t = \frac{N_{d22}^t}{f_{yd}}$$

$$a^t = \frac{N_c^t}{f_c}$$

$$A_{s1}^b = \frac{N_{d11}^b}{f_{yd}}$$

$$A_{s2}^b = \frac{N_{d22}^b}{f_{yd}}$$

$$a^b = \frac{N_c^b}{f_c}$$

where:

$$f_{yd} = \frac{f_{yk}}{\gamma_S}$$

$$f_{cd} = \alpha_{CC}\frac{f_{ck}}{\gamma_C}$$

f_{yd} = design yield strength of reinforcement
f_{yk} = characteristic yield strength of reinforcement
γ_S = partial factor for reinforcement
f_{cd} = design value of concrete compressive strength
f_{ck} = characteristic compressive cylinder strength of concrete at 28 days
α_{CC} = coefficient taking account of long-term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied
γ_C = partial factor for concrete

As the outer layer (top or bottom) of concrete is subjected to the set of forces N_{11}, N_{22}, and N_{12}, the concrete compressive strength, f_c, is computed as follows:

For uncracked concrete:
$f_c = f_{cd1} = f_{cd}$

For cracked concrete:
$f_c = f_{cd2} = \upsilon_1 f_{cd}$

where:

$\upsilon_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right)$ where f_{ck} is in MPa (EC2-1 Eq. 6.6N)

υ_1 can also be found in National Annex (Appendix B)

For design cases I, II, and III, in which the concrete is assumed to be cracked, f_c is interpolated between the values of f_{cd1} and f_{cd2} based on the model by Vecchio and Collins (1986) that the compressive strength of concrete decreases as the maximum tensile strain ε_1 increases:

$$f_c = \begin{cases} f_{cd2} & \beta < 0.6 \\ \dfrac{f_{cd1}}{0.8 - 0.34\left(\frac{\varepsilon_1}{\varepsilon_{c3}}\right)} & 0.6 \le \beta \le 1.0 \\ f_{cd1} & 1.0 < \beta \end{cases}$$

$$\beta = \frac{1.0}{0.8 - 0.34\left(\frac{\varepsilon_1}{\varepsilon_{c3}}\right)}$$

$$\varepsilon_1 = \begin{cases} 2\left(\varepsilon_{yd} - 0.5\varepsilon_{c3}\right) & \textit{Design case I} \\ \dfrac{\varepsilon_{yd} - \varepsilon_{c3}\cos^2\theta}{\sin^2\theta} & \textit{Design case II} \\ \dfrac{\varepsilon_{yd} - \varepsilon_{c3}\sin^2\theta}{\cos^2\theta} & \textit{Design case III} \end{cases}$$

where:

$\varepsilon_{yd} =$ yield strain of reinforcement

$\varepsilon_{c3} =$ strain at concrete compressive strength f_{cd}

$\theta =$ angle between 1-direction and the principal tensile direction (perpendicular to the cracks)

For design case IV, in which the concrete is uncracked, the concrete compressive stress is taken as $f_c = f_{cd1}$

The entire procedure as described in this section assumes the values of the design thickness of the top and bottom layers are known to start the design, and at the end these thicknesses are determined again. An iteration is then implemented to ensure the initial and final thicknesses are matched. The steps of the iteration are as follows:

1. Assume both thicknesses of the top and bottom layers have the value of $0.2h$

2. Calculate the forces at the center of the top and bottom layers
3. Determine the design case for each layer
4. Compute the forces to be resisted by the reinforcement at the center of each layer in each direction and the compressive force to be resisted by concrete
5. Adjust the forces for reinforcement design at the actual location of the reinforcement in each direction and each layer. For the design case and the direction in which there is only one layer of reinforcement required, determine ΔN and update the forces in the other layer if $\Delta N \neq 0$. For this layer without the reinforcement, re-determine the design case and the compressive force to be resisted by concrete.
6. Calculate the thickness of the top and bottom layers. If the difference of these thicknesses and those assumed in step 1 is within specified tolerance, the required reinforcement is calculated, and the design is done. Otherwise, the newly assumed thickness is taken as the average of these thicknesses and the previously assumed thicknesses, and the procedure is repeated from steps 2 to 6.

3.4.2 Design of Shell with Single Layer of Reinforcement

The layered-sandwich model is also applied for the design of shell element with a single layer of reinforcement. However, the reinforcement layer is assumed to be at the middle of the shell section and the design amount of reinforcement is reported as the top layer while that of the bottom layer is taken to be zero. For the design cases in which only one layer of reinforcement is required (either top or bottom, not both) or no reinforcement is required for top and bottom layers, the design is carried out as described in preceding section.

For the design case that results in both top and bottom reinforcement required, the design force in the reinforcement at its location at the middle of the shell section is determined as illustrated in Figure 3-7 for the design in direction 1.

Since reinforcement is needed for both top and bottom layers but only a single layer of reinforcement is provided at the middle of the shell section, either ΔN_{11}^t or ΔN_{11}^b must equilibrate N_{11}^t, N_{22}^t, and N_{12}^t (or N_{11}^b, N_{22}^b, and N_{12}^b) of the top or bottom layer such that after being adjusted by ΔN_{11}^t or ΔN_{11}^b the reinforcement is no longer needed for the top or bottom layer. Two scenarios are carried out to determine N_{d11}^t as follows:

1. Let $\Delta N_{11}^t = -(N_{11}^t + |N_{12}^t|) \rightarrow N_{11\_updated}^t = N_{11}^t + \Delta N_{11}^t = -|N_{12}^t|$
   $\rightarrow N_{s11\_updated}^t = N_{11\_updated}^t + |N_{12}^t| = 0$

Therefore, reinforcement is not required for the top layer. Using moment equilibrium equations to solve for the design axial force component per unit length along direction 1 at the reinforcement location (middle layer), we arrive at the following equation:

$$\sum M_{center\ of\ the\ bottom\ layer} = 0 \rightarrow N_{d11}^t = \frac{\left(N_{s11\_updated}^t - \Delta N_{11}^t\right)\left(h^t + h^b\right)}{h^b}$$

$$\sum F = 0 \rightarrow \Delta N_{11}^b = N_{s11\_updated}^t + N_{s11}^b - \Delta N_{11}^t - N_{d11}^t$$

2. Let $\Delta N_{11}^{b} = -\left(N_{11}^{b} + \left|N_{12}^{b}\right|\right) \rightarrow N_{11\_updated}^{b} = N_{11}^{b} + \Delta N_{11}^{b} = -\left|N_{12}^{b}\right|$
   $\rightarrow N_{s11\_updated}^{b} = N_{11\_updated}^{b} + \left|N_{12}^{b}\right| = 0$

Therefore, reinforcement is not required for the bottom layer. Using moment and force equilibrium equations to solve for the design axial force component per unit length along direction 1 at the reinforcement location (middle layer), we arrive at the following equation:

$$\sum M_{center\ of\ the\ top\ layer} = 0 \rightarrow N_{d11}^{t} = \frac{\left(N_{s11\_updated}^{b} - \Delta N_{11}^{b}\right)\left(h^{t} + h^{b}\right)}{h^{t}}$$
$$\sum F = 0 \rightarrow \Delta N_{11}^{t} = N_{s11}^{t} + N_{s11\_updated}^{b} - \Delta N_{11}^{b} - N_{d11}^{t}$$

The N_{d11}^{t} computed in the 2 scenarios above are compared and the case providing the smaller N_{d11}^{t} is selected as the optimal design for reinforcement.

The design force in the reinforcement along direction 2 are determined similarly.

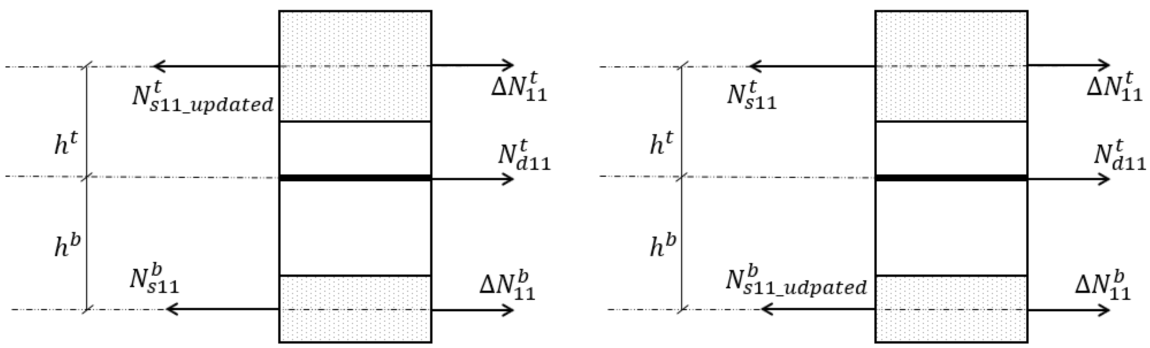

Figure 3-7: Loads on single layer of reinforcement – Left: case 1; Right: case 2

3.4.3 Design for Shear

For shear design, the shear demand in the direction of principal shear is calculated as follows:

$$V_{o} = \sqrt{V_{1}^{2} + V_{2}^{2}} \qquad \text{(EC2-2 Eq. LL.121)}$$
$$v_{o} = \frac{V_{o}}{d} = \frac{V_{o}}{h - a^{t} - a^{b}}$$
$$tan\varphi_{o} = \frac{V_{2}}{V_{1}} \qquad \text{(EC2-2 Eq. LL.122)}$$

The concrete shear strength is determined by:

$$v_{Rd,c} = C_{Rd,c}k(100\rho_{l}f_{ck})^{1/3} + k_{1}\sigma_{cp} \geq \left(v_{min} + k_{1}\sigma_{cp}\right) \qquad \text{(EC2-1 Eq. 6.2.a \& 6.2.b)}$$

where:

$f_{ck} =$ concrete strength in MPa.

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$ with d in mm.

$\rho_{l} = \rho_{l1}cos^{2}\varphi_{o} + \rho_{l2}sin^{2}\varphi_{o} \leq 0.02$ (EC2-2 Eq. LL.123)

$$\rho_{l1} = \frac{A_{s1}^t + A_{s1}^b}{d}$$
$$\rho_{l2} = \frac{A_{s2}^t + A_{s2}^b}{d}$$
$$\sigma_{cp} = \sigma_{cp1} cos^2\varphi_o + \sigma_{cp2} sin^2\varphi_o$$
$$\sigma_{cp1} = \frac{N_{11}}{d} < 0.2 f_{cd}$$
$$\sigma_{cp2} = \frac{N_{22}}{d} < 0.2 f_{cd}$$
$$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2} \qquad \text{(EC2-1 Eq. 6.3N)}$$
$$d = d_1 cos^2\varphi_o + d_2 sin^2\varphi_o$$
$$d_1 = h - max\{c_1^{t\prime}, 0.5a^t\} - max\{c_1^{b\prime}, 0.5a^b\}$$
$$d_2 = h - max\{c_2^{t\prime}, 0.5a^t\} - max\{c_2^{b\prime}, 0.5a^b\}$$
$$c_1^{t\prime} = \begin{cases} c_1^t & A_{s1}^t > 0 \\ 0 & A_{s1}^t = 0 \end{cases}$$
$$c_1^{b\prime} = \begin{cases} c_1^b & A_{s1}^b > 0 \\ 0 & A_{s1}^b = 0 \end{cases}$$
$$c_2^{t\prime} = \begin{cases} c_2^t & A_{s2}^t > 0 \\ 0 & A_{s2}^t = 0 \end{cases}$$
$$c_2^{b\prime} = \begin{cases} c_2^b & A_{s2}^b > 0 \\ 0 & A_{s2}^b = 0 \end{cases}$$

c_1^t and c_2^t are the concrete cover measured from the top of the section to the center of top reinforcement in directions 1 and 2, respectively. And c_1^b and c_2^b the concrete cover measured from the bottom of the section to the center of bottom reinforcement in directions 1 and 2, respectively. For shell element with single layer of reinforcement, as the reinforcement layer is assumed to be at the middle of the shell section and the design amount of reinforcement is reported as the top layer, c_1^t and c_2^t are both taken as half of the thickness of the shell section while c_1^b and c_2^b are both taken to be zero in calculation of d_1 and d_2.

The values of $C_{Rd,c}$, v_{min}, and k_1 are found in National Annex as described in Appendix B.

If $v_o \leq v_{Rd,c}$, the concrete shear capacity is sufficient and transverse reinforcement is not required. Otherwise, the longitudinal reinforcement in the direction of principal shear, ρ_l, will be increased to ρ_{lv} such that:

$$v_o = v_{Rd,c} = C_{Rd,c} k (100 \rho_{lv} f_{ck})^{1/3} + k_1 \sigma_{cp} \qquad \text{(EC2-1 Eq. 6.2.a \& 6.2.b)}$$

ρ_{lv} can be back-calculated as:

$$\rho_{lv} = \frac{1}{100 f_{ck}} \left(\frac{v_o - k_1 \sigma_{cp}}{C_{Rd,c} k} \right)^3 \leq 0.02$$

If $\rho_{lv} \leq 0.02$, meaning the concrete shear capacity is sufficient to resist shear demand by increasing longitudinal reinforcement, then all top and bottom longitudinal reinforcement in directions 1 and 2 will be increased by the ratio $\frac{\rho_{lv}}{\rho_l}$. For the design case in which longitudinal reinforcement is not required in the outer layers in both directions, resulting in $\rho_l = 0$, transverse reinforcement will be determined as described subsequently.

Otherwise, if $\rho_{lv} > 0.02$, the transverse reinforcement will be provided, and longitudinal reinforcement in directions 1 and 2 is re-determined to account for the increase of the membrane forces as follows:

$$N_{11v} = \frac{V_1^2}{V_o} cot\theta$$

$$N_{22v} = \frac{V_2^2}{V_o} cot\theta$$

$$N_{12v} = \frac{V_1 V_2}{V_o} cot\theta$$

$$N_{11}^t = \frac{(N_{11} + N_{11v})h^b - M_{11}}{h^t + h^b}$$

$$N_{22}^t = \frac{(N_{22} + N_{22v})h^b - M_{22}}{h^t + h^b}$$

$$N_{12}^t = \frac{(N_{12} + N_{12v})h^b - M_{12}}{h^t + h^b}$$

$$N_{11}^b = \frac{(N_{11} + N_{11v})h^t + M_{11}}{h^t + h^b}$$

$$N_{22}^b = \frac{(N_{22} + N_{22v})h^t + M_{22}}{h^t + h^b}$$

$$N_{12}^b = \frac{(N_{12} + N_{12v})h^t + M_{12}}{h^t + h^b}$$

where θ is the angle of the inclined concrete compression strut. It is taken as 45-degree by default and can be modified in the Preference form by the user.

The design for longitudinal reinforcement to resist the updated membrane forces is carried out. As the EC2 code alllows this increase of membrane forces be subtituted by a shift in the longitudinal reinforcement as described in EC2 9.2.1.3(2), an option to whether or not include this additional tensile force is available in the Preference form.

It is worth noting that for load case and/or combo with multi-steps, any load step that results in insufficient concrete shear capacity even though longitudinal reinforcement has been increased, and transverse reinforcement is required, all other load steps will be re-designed with the assumption that the transverse reinforcement is to be provided.

The transverse reinforcement is determined by:

$$\frac{A_{sw}}{s} = \frac{V_o}{z f_{yd} cot\theta} \qquad \text{(EC2-1 Eq. 6.8)}$$

To prevent crushing of the concrete compression struts, the shear resistance is limited by the maximum value of $V_{Rd,max}$:

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{cd} \frac{1}{(cot\theta + tan\theta)} \qquad \text{(EC2-1 Eq. 6.9)}$$

where:

$z = d$
$b_w = 1$ as the design is performed over a unit of shell element
f_{yd} = design yield strength of reinforcement
$\alpha_{cw} = 1$ to account for the state of stress in the compression chord
$\upsilon_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right)$ where f_{ck} is in MPa (EC2-1 Eq. 6.6N)
α_{cw} and υ_1 can also be found in National Annex (Appendix B)

The required amount of transverse reinforcement in the principal shear direction, $\frac{A_{sw}}{s}$, is provided by the transverse reinforcement in the directions 1 and 2, $\frac{A_{sw1}}{s}$ and $\frac{A_{sw2}}{s}$, respectively, through the following relations:

$$\begin{cases} \left(\frac{A_{sw}}{s}\right) = \left(\frac{A_{sw1}}{s}\right) cos^2\varphi_o + \left(\frac{A_{sw2}}{s}\right) sin^2\varphi_o \\ \frac{\left(\frac{A_{sw1}}{s}\right)}{\left(\frac{A_{sw2}}{s}\right)} = \left|\frac{V_1}{V_2}\right| \end{cases}$$

Resulting in:

$$\begin{cases} \left(\frac{A_{sw1}}{s}\right) = \left(\frac{A_{sw}}{s}\right) \frac{1}{cos^2\varphi_o + \left|\frac{V_2}{V_1}\right| sin^2\varphi_o} \\ \left(\frac{A_{sw2}}{s}\right) = \left|\frac{V_2}{V_1}\right| \left(\frac{A_{sw1}}{s}\right) \end{cases}$$

APPENDICES

Appendix A
References

Brondum-Nielsen, T., (1974). *Optimum Design of Reinforced Concrete Shells and Slabs*, Technical University of Denmark, Department of Civil Engineering.

Colombo, A.B., J. C. Della Bella, and T. N. Bittencourt (2014). "An algorithm for the automatic design of concrete shell reinforcement," *IBRACON Struct. Mater. J.*, vol. 7, no. 1, pp. 53–67.

Craveiro, M.V., T.N. Bittencourt, J.C.D. Bella (2021). "Design and Verification of Reinforced Concrete Shell Elements," *Rev. IBRACON Estrut. Mater.*, vol. 14, no. 3, e14305, 2021, https://doi.org/10.1590/S1983-41952021000300005

EN 1992-1-1:2004 (2004). *Eurocode 2 – Design of Concrete Structures – Part 1-1: General Rules and Rules for Buildings*, European Committee for Standardization, rue de Stassart, 36, B-1050 Brussels, Belgium.

EN 1992-2:2005 (2005). *Eurocode 2 – Design of Concrete Structures – Part 2: Concrete bridges – Design and detailing rules*, European Committee for Standardization, rue de Stassart, 36, B-1050 Brussels, Belgium.

Marti, P. (1990). "Design of Concrete Slabs for Transverse Shear," *ACI Journal*, pp. 180-190.

Vecchio, F., and M.P. Collins (1986). "The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear," *ACI Journal*, pp. 219-231.

Appendix B
Nationally Determined Parameters (NDPs)

This appendix provides a listing of the Nationally Determined Parameters (NDPs) used by default for the various country implementations. Several of these parameters can be modified through the design preferences.

Table B-1 CEN Default NDPs

NDP	Clause	Value
γ_C	2.4.2.4(1)	1.5
γ_S	2.4.2.4(1)	1.15
α_{CC}	3.1.6(1)	1.0
Max f_{yk}	3.2.2(3)	600MPa
$C_{Rd,c}$	6.2.2(1)	$0.18/\gamma_C$
v_{min}	6.2.2(1)	$0.035k^{3/2}f_{ck}^{1/2}$
k_1	6.2.2(1)	0.15
$Cot\theta$	6.2.3(2)	$1.0 \leq Cot\theta \leq 2.5$
v_1	6.2.3(3)	$0.6\left(1-\frac{f_{ck}}{250}\right)$

Table B-2 United Kingdom NDPs

NDP	Clause	Value
α_{CC}	3.1.6(1)	0.85

Table B-3 Slovenia NDPs

All parameters for Slovenia concrete shell design are as those by CEN Default.

Table B-4 Norway NDPs

NDP	Clause	Value
α_{CC}	3.1.6(1)	0.85
k_1	6.2.2(1)	0.15 for compression 0.3 for tension

Table B-5 Singapore NDPs

NDP	Clause	Value
α_{CC}	3.1.6(1)	0.85

Table B-6 Sweden NDPs

All parameters for Sweden concrete shell design are as those by CEN Default.

Table B-7 Finland NDPs

NDP	Clause	Value
α_{CC}	3.1.6(1)	0.85
Max f_{yk}	3.2.2(3)	700MPa

Table B-8 Denmark NDPs

NDP	Clause	Value
γ_C	2.4.2.4(1)	1.45
γ_S	2.4.2.4(1)	1.20
Max f_{yk}	3.2.2(3)	650MPa

Table B-9 Portugal NDPs

NDP	Clause	Value
Max f_{yk}	3.2.2(3)	500MPa

Table B-10 Germany NDPs

NDP	Clause	Value
α_{CC}	3.1.6(1)	0.85
Max f_{yk}	3.2.2(3)	500MPa
$C_{Rd,c}$	6.2.2(1)	$0.15/\gamma_C$
v_{min}	6.2.2(1)	$(0.0525/\gamma_C)k^{3/2}f_{ck}^{1/2}$ for $d \leq 600mm$ $(0.0375/\gamma_C)k^{3/2}f_{ck}^{1/2}$ for $d > 800mm$
k_1	6.2.2(1)	0.12
$Cot\theta$	6.2.3(2)	$1.0 \leq Cot\theta \leq 3.0$
v_1	6.2.3(3)	$0.75v_2$ where $v_2 = (1.1 - \frac{f_{ck}}{500}) \leq 1.0$

Table B-11 Poland NDPs

NDP	Clause	Value
γ_C	2.4.2.4(1)	1.4
$Cot\theta$	6.2.3(2)	$1.0 \leq Cot\theta \leq 2.0$

Table B-12 Ireland NDPs

NDP	Clause	Value
α_{CC}	3.1.6(1)	0.85